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# *The Towing of Bodies in a Stratified Fluid*

Technical Report EM-71-1

JAMES W. DAILY  
WALTER DEBLER

November 1971

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THE TOWING OF BODIES IN A STRATIFIED FLUID

Walter Debler

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Under the Direction of  
James W. Daily  
Walter Debler

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## TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS	v
ABSTRACT	1
INTRODUCTION AND EQUIPMENT	2
EXPERIMENTS WITH A CIRCULAR CYLINDER	9
EXPERIMENTS WITH A VERTICAL PLATE AND A SPHERE	19
CONCLUSIONS	21
REFERENCES	24
ACKNOWLEDGEMENTS	25

## LIST OF ILLUSTRATIONS

	Page
Fig. 1 a. Side view of towing tank, drive and filling appurtenances	2
Fig. 1 b. Top view of towing tank showing flow distributor for filling.	3
Fig. 2 a. Cylinder and support struts.	7
Fig. 2 b. Sphere and plate used in experiments.	7
Fig. 3 a. Translating cylinder ( $R = 11$ , $F_d = 0.0455$ ) with a following fluid train.	12
Fig. 3 b. Translating cylinder ( $R = 5$ , $F_d = 0.0636$ ) 5 minutes after starting.	12
Fig. 3 c. Same cylinder 15 minutes after starting. Fluid which was originally behind cylinder (i. e. to the right of horizontal wedge) is being left behind.	12
Fig. 4 a. Elongated, stable vortex pair upstream of cylinder which is moving to the right. Downstream pattern, a wedge formation is associated with downstream blocking. $R = 13$ and $F_d = 0.072$ in this experiment appear to be marginal for downstream blocking.	15
Fig. 4 b. Upstream vortices without downstream blocking. Horizontal lines at the right caused by dye coming from cylinder. Lower line due to gradual sinking of dye. No significant lee waves present. Symmetric structure of upstream vortices indicated. $R = 47$ and $F_d = 0.189$ .	15
Fig. 4 c. View of dye from cylinder moving ahead of cylinder to delineate ultimately a vortex pair $R = 40$ and $F_d = 0.16$ .	15
Fig. 5 a. Upstream vortices and a small lee waves. $R = 77$ and $F_d = 0.31$ .	16

		Page
Fig. 5 b.	Cylinder free of upstream and downstream vortices. Lee waves present. $R = 62$ and $F_d = 0.243$ .	16
Fig. 5 c.	Cylinder with attached, stable, downstream vortices with lee waves. $R = 160$ and $F_d = 0.641$ .	16
Fig. 5 d.	Karman vortex street decaying downstream of cylinder. $R = 444$ and $F_d = 1.73$ .	16
Fig. 6	Flow phenomena diagram for two dimensional circular cylinder translating in a stratified fluid. Vertical coordinate is ratio of the buoyant and viscous forces.	17
Fig. 7.	Vertical plate moving toward the left. $R = 93$ and $F_d = 0.037$ . Dye sheets in central channel show upstream and downstream blocking.	19
Fig. 8.	Sphere being towed in a stratified liquid $R = 17.6$ and $F_d = 0.037$ . Wake appears two dimensional in view. Extent of latitudes having horizontal flow can be inferred.	20
Fig. 9.	Four stages in the development of a two dimensional Karman vortex street behind a sphere moving in a stratified liquid. $R = 3777$ and $F_d = 0.8$ .	22

### Abstract

A series of experiments were conducted in water which was linearly stratified by the addition of prescribed amounts of sodium chloride to the layers of water that were carefully added to the towing tank. A circular cylinder with its axis horizontal, and perpendicular to the direction of motion, was tested as well as a vertical plate and a sphere. The experiments were intended to catalogue the kinds of flow that would be generated in various density gradients as the body's Reynolds number was increased from values starting with 5. (At low Reynolds numbers for the cylinder a forward or "nose" vortex pair was observed and no lee waves occurred. At very low Reynolds and densimetric Froude number, the liquid did not flow over and behind the cylinder. Rather blocking occurred fore and aft of the cylinder. With increased Reynolds numbers, the forward vortices decreased in length and lee waves occurred. The forward vortices disappeared with larger Reynolds number and the lee waves continued. A Reynolds number was reached at which a stable vortex pair was attached aft of the body. Ultimately, with higher Reynolds numbers, the vortex pair became unstable and a Karman vortex street ensued which also was accompanied by lee waves. Similar, but more limited results, were observed for the vertical plate that was towed. While translating a sphere through a stratified fluid a condition was achieved which resulted in an apparent two-dimensional Karman vortex street (with the vortices' axes vertical). The fluid preferred to flow around the sphere instead of over and under it. As the sphere's Reynolds number was increased, the latitudes, above and below the equator, where the water flowed around the sphere decreased.

## Introduction and Equipment

Examples of the flow past bodies in a stratified fluid are cited in Yih (1965). They range from the "blocking" upstream and downstream of a vertical object to lee waves and rollers. Still the question remained concerning the flow conditions under which "blocking" would give way to lee waves. Also what must be the circumstances for viscous manifestations such as the Karman vortex street to appear? The experiments that will be described in this paper were undertaken to explore these phenomena with the intent of categorizing the fluid motions associated with the flow past bodies in a stratified fluid.

The experiments were carried out in a tank which resembled that described in Yih (1959); however, the size was larger and a more elaborate towing scheme was employed. The tank and filling contraptions are shown in figure 1 a. The box-like structure at the left simply raised the filling bottle to permit gravity feeding of the tank. Water in which a prescribed amount of sodium chloride was mixed was added to the bottle, from whence it flowed through the tube that is supported by the low "box" at the top and center of the tank. The saline solution was thus admitted to the flow distributor which is partially visible just below the horizontal meter stick. Figure 1 b shows a top view of the tank. The flow distributor can be seen in the middle portion of the tank. It looks something like a barge with folded, brass sheet for the bottom and sides. Expandable and replaceable aluminum pontoons, actually aluminum foil baking forms, kept the barge afloat with a draft of about 1 centimeter. The barge's bow and stern were porous plastic foam which caused the liquid that was being added to the tank to seep slowly through it without jets and eddies.

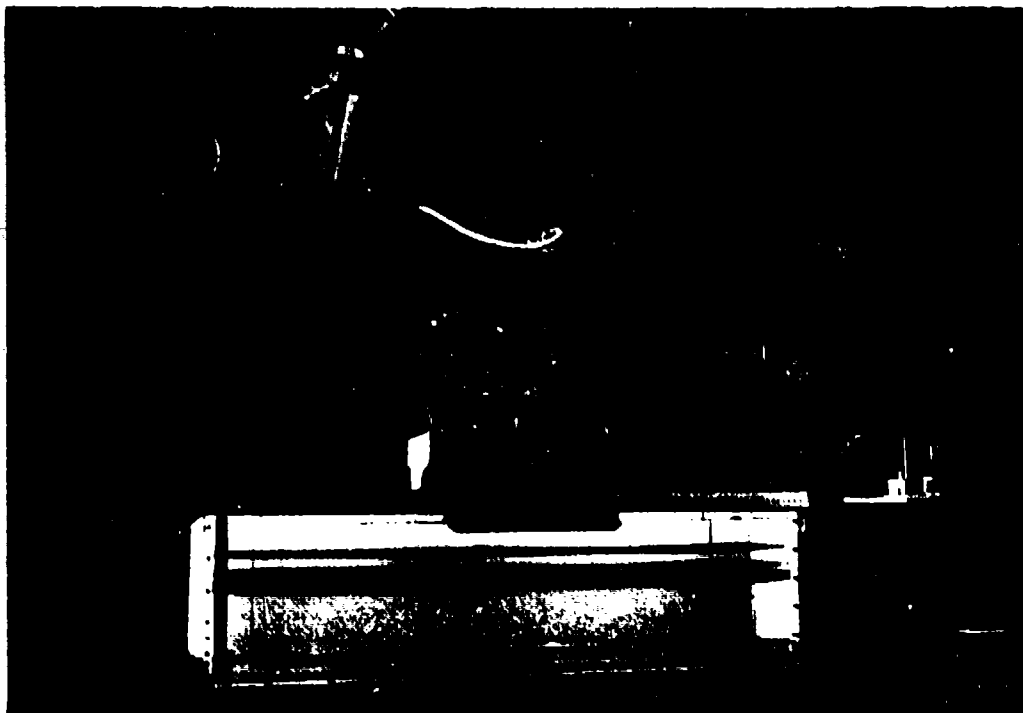


Fig. 1 a. Side view of towing tank, drive and filling appurtenances.

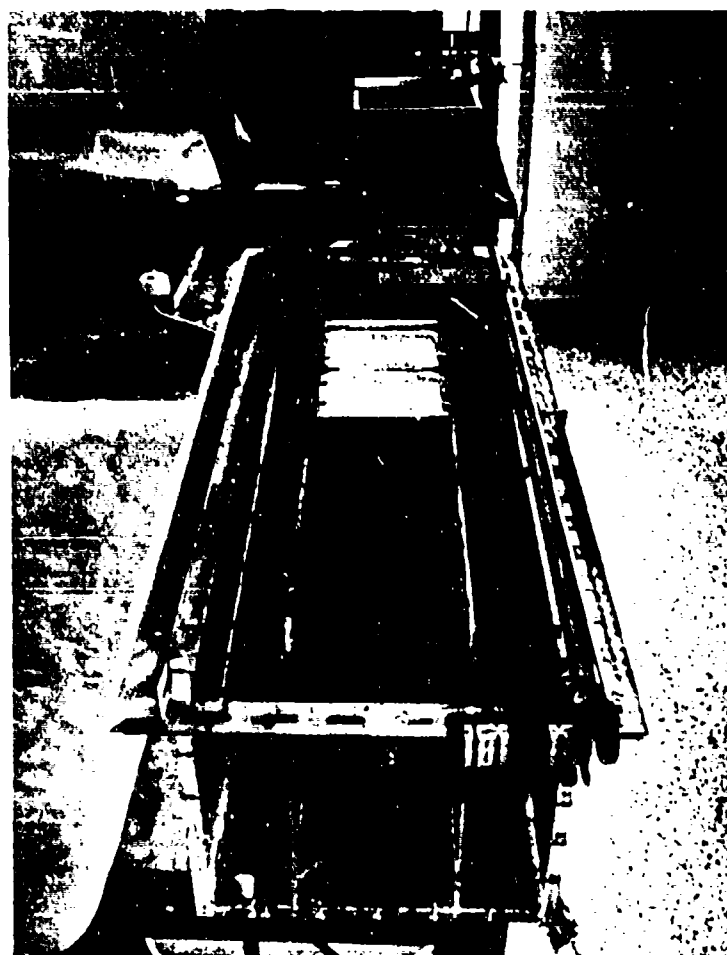


Fig. 1 b. Top view of towing tank showing flow distributor for filling.



The salt water flowed into the middle of the barge between the 2 pontoons from a rigid tube; it then flowed around the pontoons and uniformly through the plastic foam fore and aft. Before an object was towed, the flow distributor was carefully removed from the tank. The layers that were added were 1 cm. deep and 25 layers were added to the tank. The water that was used was stored in the laboratory for about 12 hours prior to filling to encourage thermal equilibrium with the ambient conditions. The tank was filled in four to five hours and allowed to rest overnight to permit diffusion to effect a linear and continuous gradient. No direct measurements of the density gradient were made, but the author's previous experience (Debler (1959)) would indicate that with the precautions that were taken a linear density variation could be expected.

Figure 1 b also shows the manner in which the tank was divided. The center section was about 14 cm. wide and the two side channels were each 7 cm. wide. The reason for these three passages is to allow a symmetric return flow along the two sides in the event that the cylinder, which would span the center section, should push a layer of fluid ahead of it (cf. Yih (1959)). The tank was 1 meter long. The false interior walls were adjusted by screw supports so that they were parallel and aligned with the towing mechanism. The towing carriage had three wheels. The two on the right hand side of figure 1 b were grooved to ride atop the slotted angle that acted as a brace for the tank and a rail for the carriage. The third wheel, on the left, was not grooved so as to allow for possible misalignment of two pieces of slotted angle that served as the rails. The motor and gear train that formed the power unit were from a German construction toy, (Fischer-Werk, Tumlingen Kreis Freudenstadt) and proved to have a satisfactory speed range and constant towing speed. The towing string was wound upon a spool with helical grooves. The string went from this spool

over, and under, the nearest grooved wheel on the carriage. From there it went back to the end of the tank where it was secured, resting atop the rail. A second cord was attached to the top of the rail at the near end shown in figure 1 b. The cord went toward the carriage, along the rail, under and over the first grooved wheel and then back toward the end where it was secured. When it reached this end it passed over a pulley and, by virtue of the weight that was attached to it, hung vertically. Thus as the motor's drum gathered the cord, the cord rolled over the wheel closest to the motor and rotated it causing the carriage to translate. The drive motor had to accomplish this while raising the weight that was attached to the second cord which served to eliminate any backlash in the system. When the motor was run in the opposite direction, and cord was let out, the weight at the opposite end was allowed thereby to descend and pull the carriage toward the end away from the drive motor. The cords could have been attached directly to the carriage but it was found that using the available power to effect a rotation of the wheels and convert that motion to a translation was superior to translating directly the carriage and causing the wheels to rotate. Considerable time was spent to be certain that the drive system produced a smooth and continuous translation of the carriage. This state was determined by visually observing the moving cylinder and noting the absence of any stick-slip and other erratic motion. With the available gear train, rheostat and two winding spools translation speeds between 0.02 and 2.5 cm./sec. were conveniently attained.

A plastic cylinder (cf. figure 2a) with a diameter of 2 cm. was suspended by two clear, plastic strips, 1 mm. in thickness. These vertical supports were located just inside the false inner walls which were adjusted so that only a slight clearance existed between them and these transparent struts. The plate with a height of 2 cm. and the sphere with a diameter of 3.77 cm., a ping-pong ball, are shown in figure 2 b.

The fluid motion was observed by dyeing the fluid. During the filling process certain layers were dyed with small amounts of nigrosin before admission into the tank. These dyed layers did not diffuse too much between the time of the filling and the experiment and were particularly suitable to observing wave motion. Just prior to the experiment vertical dye streaks were added by dropping fine crystals of potassium permanganate into the water. In order to increase the observed color contrast, the crystals were dropped in a line parallel to the width of the tank. In this way one observed from the side of the tank the edge of a vertical sheet of dye. Such dye "curtains" could be added at any number of desired stations along the tank, both in the center section or the side passages. In addition, dye crystal were dropped from directly above the object to be towed. It was hoped that some of these would land on the body. A few minutes were allowed to elapse during which the dye crystals on the body dissolved and formed a colored solution that showed

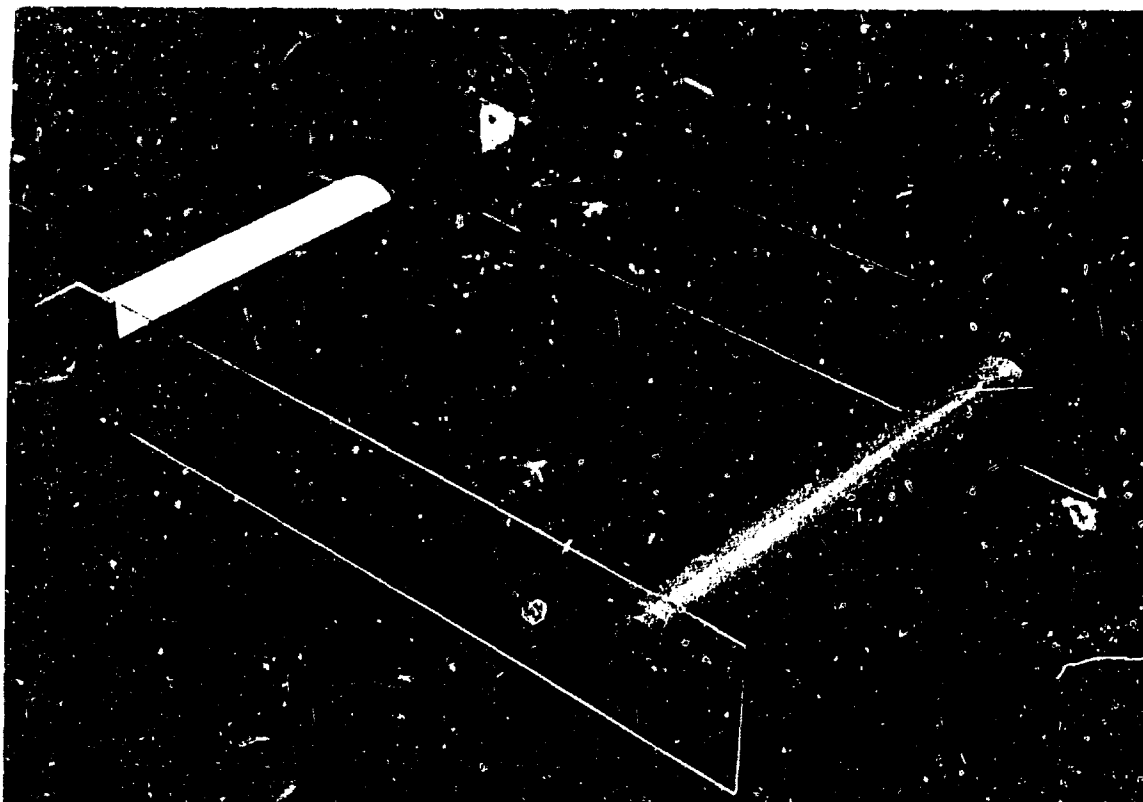


Fig. 2 a. Cylinder and support struts.

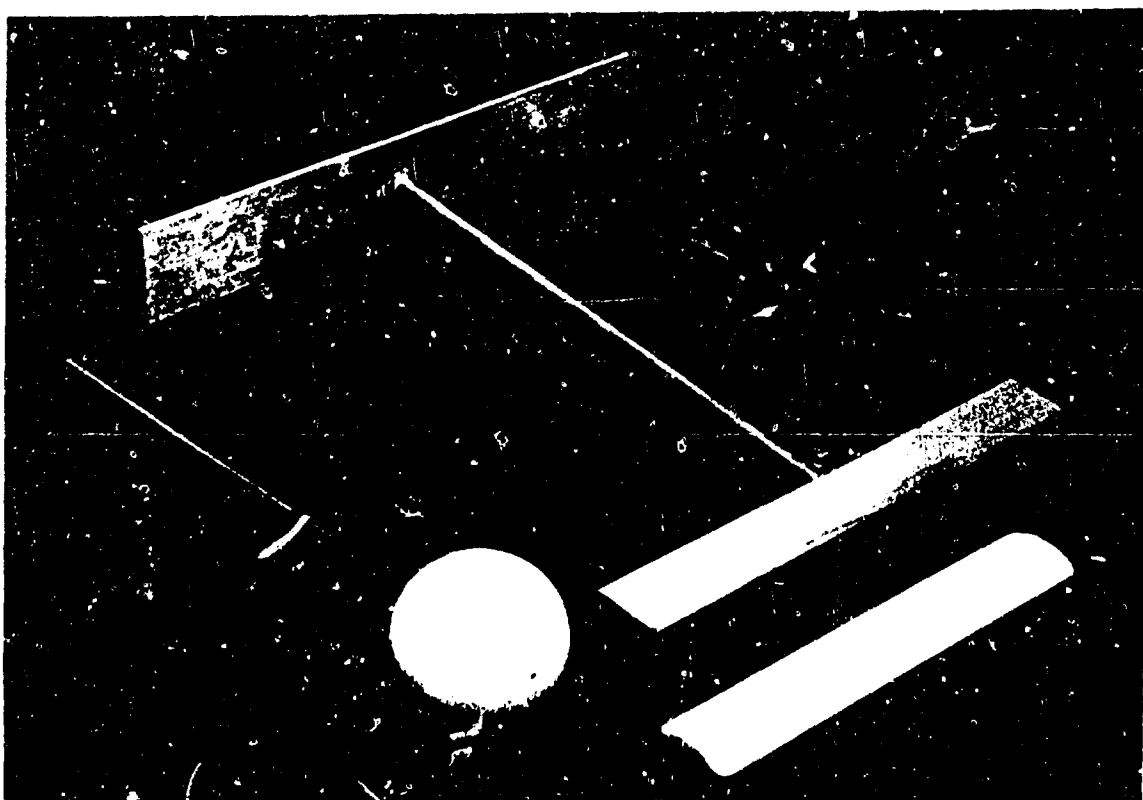


Fig. 2 b. Sphere and plate used in experiments.

the motion next to the solid boundary. While the object was being towed the crystals remained on its surface and continued to dissolve and indicate the flow. Quite naturally some of the crystals did not land on the body or rolled off and descended to the bottom. In so doing they formed a dye "curtain" at the body and thereby delineated the fluid in the center channel that was initially behind the body. This proved to be unexpectedly useful later when the photographs were analyzed. Dye streaks could be added to the outside channels of the tank as well as the center one in which the body moved. This permitted the observation of induced flows at considerable distances from the body. The tank was illuminated from the rear with two incandescent lamps. These shone through a piece of translucent paper which diffused the light.

#### Experiments with a Circular Cylinder

Density differences between the top and bottom layers of 0.01, 0.004, 0.002 and 0.001 gm./cc. were used. This gave a density gradient between  $0.4 \text{ mg./cm.}^4$  and  $0.04 \text{ mg./cm.}^4$ . At this lowest gradient one could detect minor effects due to small temperature differences between the plastic tank and the liquid being added. A slight curvature of the dyed layers could be seen near the tank's outside walls. No flow effects are believed to have resulted from this occasional lack of complete temperature equilibrium of the liquid and its container.

After the desired dye curtains had been set, the rheostat was slowly turned to a predetermined position. This was usually the slowest speed for the test series planned for the particular filling of the tank. It was intended thereby to minimize the disturbance to the density gradient and permit more than one test to be run with the particular solution. If an interesting flow pattern occurred during one of these secondary tests, an experiment was planned for the next day which duplicated the test parameters and was the first one

of that day's series. Simultaneously with the initiation of towing a stop-watch was started. Photographs were taken of the entire tank as well as close up pictures of the body during a tow. The meter stick and watch appeared in each of the photographs of the entire tank. Thus an estimate of the body's speed could be found from the photographs. However, sufficient parallax was present to introduce some uncertainty about the displacements measured from enlargements of the negatives. Fortunately displacements and time intervals were recorded directly during the course of the experiments. From three to five such observations were recorded during a run so that the speed could be determined reliably. After the body had been moved the length of the tank, the test was terminated. The drive motor was shut off and the liquid in the tank allowed to come to a complete rest during a 30 minute pause. The absence of residual currents could be assured if the dye curtains that were prepared just before the next test remained motionless.

The first tests were run at some of the lowest speeds possible and gave cylinder Reynolds numbers between five and fifty. It was anticipated that the fluid directly in front of the cylinder would be pushed forward by the cylinder by blocking as exemplified by Yih (1959). The slug-like motion of the liquid behind the cylinder and at the same level was awaited also. This situation was realized in some of the experiments. Figure 3 a shows the cylinder after it has moved about two thirds of the tank's length. The density gradient was the strongest employed,  $0.4 \text{ mg./cm.}^2$ , the speed was  $0.055 \text{ cm./sec.}$  This gives a Reynolds number of 11 and a densimetric Froude number,  $U/\sqrt{\frac{\Delta\rho g D^2}{d \rho}}$ , of 0.046. In this definition of  $F_d$   $\rho$  is the density,  $d$  is the depth of liquid so that  $\Delta\rho/d$  is the density gradient,  $D$  is the cylinder diameter and  $U$  is the speed of translation. About 20 minutes have elapsed since the cylinder was started. One sees a long tail behind the cylinder. This is the result of the fluid that was initially at the same level of the

cylinder, but behind it, moving forward with the speed of the cylinder. This caused the dye curtain at the cylinder's initial position to be distended horizontally and remain in contact with the cylinder. The dye that was placed on the cylinder has dissolved and appears to trail behind the cylinder and below the wedge of following fluid. There is also a trace of dye on the forward surface of the cylinder. This phenomenon will be discussed later. Earlier photographs showed the dye curtains that were considerably ahead of the cylinder in the center channel to be distorted symmetrically while the cylinder was quite distant from them. The dye curtain in the side channel that was nearest the camera was distorted with the peak pointed toward the right. This showed that the cylinder was inducing a flow in the side channels at the level of the cylinder. In this case it was the flow of liquid that was being drawn directly behind the cylinder. Viscosity tended to diffuse any velocity discontinuities so that even slower flows were induced in the fluid layers above and below the elevation of the cylinder.

However, these occurrences were not always the case. Figure 3 b and 3 c show the flow after 5 and 15 minutes, respectively. The cylinder speed was 0.025 cm./sec., the Reynolds number was 5 and the densimetric Froude number was 0.064. In this pair of pictures the speed was about one half that in the previous picture and the density gradient was larger by a factor of 10. Again, but with slightly more difficulty, one sees the sheets of dye being distended ahead of the cylinder in the center channel and the dye curtain in the side channel can be discerned, on an excellent photographic print, to be moving to the right. This would suggest that there was a flow following the cylinder. But figure 3 c clearly shows that the cylinder has outdistanced the dye curtain that originally demarcated the fluid that was fore and aft of the cylinder. Hence fluid flowed over and behind the

cylinder. The dye on the cylinder was swept along into the fine horizontal line that appears in the picture. There was no flow separation on the body. Again a wedge-like region of dye can be seen on the forward side of the cylinder.

In both of the experiments that have been described so far there was evidence that the cylinder pushed the liquid ahead of it. Had the cylinder been at rest, and the fluid streaming by it, one would expect an extensive stagnant region directly in front of the body. This is upstream blocking. The existence of such a comparable stagnant region behind the cylinder would be expected for the case shown in figure 3 a but not for the case in figure 3 c. It appears that upstream blocking does not go hand-in-hand with downstream blocking. If stratified fluid flows over and behind a two dimensional cylinder it displaces the stratified fluid that was originally behind the body. This may result in a density distribution directly behind the body, depending on the manner of flow, which is different from the fluid that was displaced. It is anticipated that in most cases these two density distributions would be such that the displaced fluid would try to intrude back into the region from which it had been expelled, i. e., toward the body. The speed of this intrusion would be governed by viscosity, the difference between the two density distributions and, perhaps, the body size. It is conjectured that if the speed of intrusion is greater than that of the moving body, downstream blocking will occur and the original downstream fluid will continue to occupy the layer behind the cylinder. Similarly, if the intrusion speed is less than the cylinder's, downstream blocking will not occur and the fluid originally aft of the body will be swept downstream. The photographs of the experiments that were conducted were examined to detect the lowest Reynolds and associated Froude numbers where downstream blocking was observed to occur. Table I summarizes the limited results in which



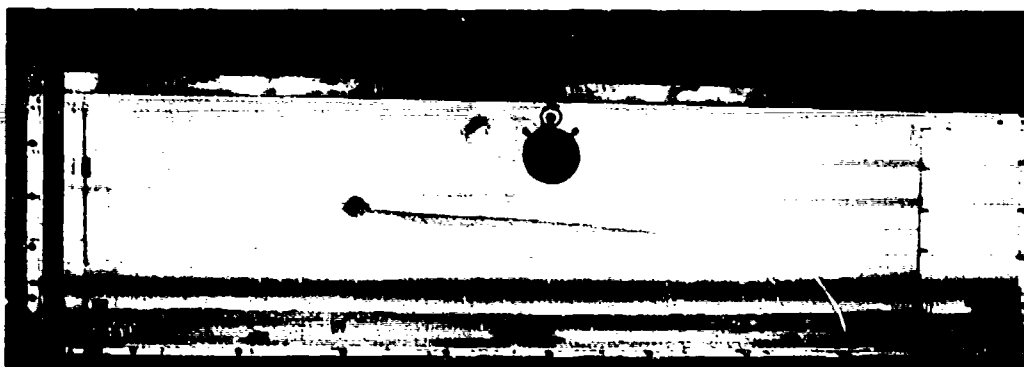


Fig. 3 a. Translating cylinder ( $R = 11$ ,  $F_d = 0.0455$ ) with a following fluid train.

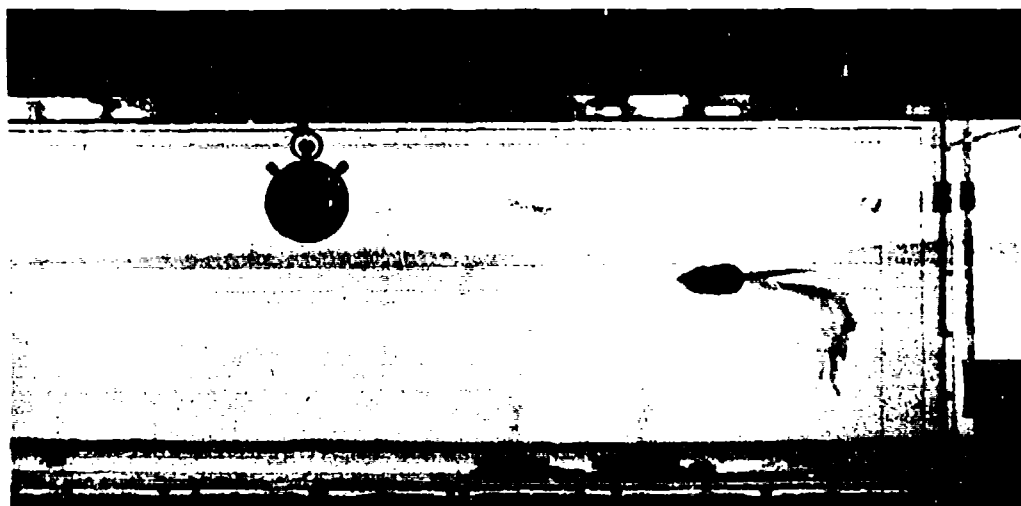


Fig. 3 b. Translating cylinder ( $R = 5$ ,  $F_d = 0.0636$ ) 5 minutes after starting.



Fig. 3 c. Same cylinder 15 minutes after starting. Fluid which was originally behind cylinder (i. e. to the right of horizontal wedge) is being left behind.

the ratio of the buoyant and viscous forces,  $R/F_d^2$ , has been included.

Table I  
Observations of Downstream Blocking

Test	Downstream Blocking	$R$	$F_d$	$R/F_d^2$	$\Delta\rho/d$
1. Cylinder	Yes	11	0.0455	5320	0.01/25
2. Cylinder	Border Line	18	0.072	3472	0.01/25
3. Cylinder	Border Line	19	0.0746	3410	0.01/25
4. Cylinder	No	26	0.1054	1800	0.01/25
5. Cylinder	No	17	0.1093	1420	0.004/25
6. Cylinder	Border Line	11	0.0983	1140	0.002/25
7. Cylinder	No	5	0.0636	1232	0.001/25
8. Cylinder	No	11.2	0.1414	560	0.001/25
9. Plate	Yes	9.3	0.037	6820	0.01/25
10. Plate	Border Line	27	0.108	2300	0.01/25
11. Plate	No	69	0.275	910	0.01/25

The table suggests that one might well concentrate on the  $R/F_d^2$  region between 1000 and 5000 to specify the conditions under which downstream blocking ceases.

When downstream blocking does not occur a host of flow configurations can be seen depending on the Reynolds and densimetric Froude numbers. However, firstly, a phenomenon that occurs at low  $R$  and  $F_d$ , regardless whether downstream blocking is present or not, will be mentioned. It has been noted that in figures 3 a and 3 c a wedge of dye can be seen ahead of the cylinder. This dye came from that which was initially placed on top of the cylinder. This means that immediately in front of the cylinder there is a fluid velocity that moves particles away from the cylinder in the direction of the cylinder's translational motion. But this is not simply upstream blocking. It is a vortex pair

that is symmetric about the cylinder's horizontal diameter. The nature and possible extent of this vortex pair can be seen from figures 4 a and 4 b. The second of these photographs shows a rather dark horizontal line emanating from the cylinder. Here the dye, which has been swept forward along the cylinder's surface, has maximum concentration. This line shows the symmetry of the vortex pair, termed in this paper as nose vortices. Figure 4 c presents an oblique view of the dye jetting ahead of the cylinder.

When downstream blocking exists lee waves do not appear. After upstream fluid flows over the cylinder and plunges behind it, perhaps overshoots and oscillates, we expect that finally viscosity will damp out any vertical motions. The experiments that were conducted bear this out. Furthermore, the nose vortices decrease in length as the cylinder increases speed in a given fluid with prescribed density gradient. Ultimately, they vanish and they can do so without the appearance of an attached vortex pair behind the cylinder. The statement of Graebel (1969) who analyzed the slow motion past bodies in a stratified fluids is fitting here. "Upstream of the body, the (relatively) weak forces of viscosity act to slowly raise a fluid particle against the action of gravity so that it can just clear the body. Once past the body, gravity will be the principal controlling factor, and the fluid particle is quickly lowered back to its original level of static equilibrium." With even greater speeds a pair of vortices occur on the downstream side of the body. They are stable and remain fixed to the body. The remainder of the flow behind the body consists of lee waves. Further increases of the cylinder's speed destabilizes the vortices and a Karman vortex street issues. The experimentation did not examine the shedding phenomenon and associated Strouhal frequencies. Some work along these lines has been done by Pao et al (1968). Figure 5 shows a series of photographs which categorize the kinds of flow fields that can be

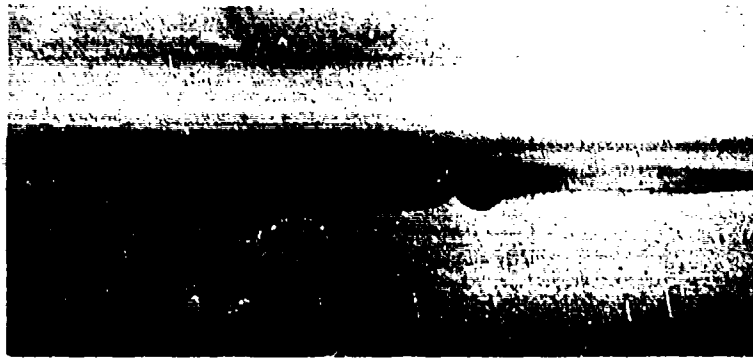


Fig. 4 a. Elongated, stable vortex pair upstream of cylinder which is moving to the right. Downstream pattern, a wedge formation, is associated with downstream blocking.  $R = 18$  and  $F_d = 0.072$  in this experiment appear to be marginal for downstream blocking.



Fig. 4 b. Upstream vortices without downstream blocking. Horizontal lines at the right caused by dye coming from cylinder. Lower line due to gradual sinking of dye. No significant lee waves present. Symmetric structure of upstream vortices indicated.  $R = 47$  and  $F_d = 0.188$ .

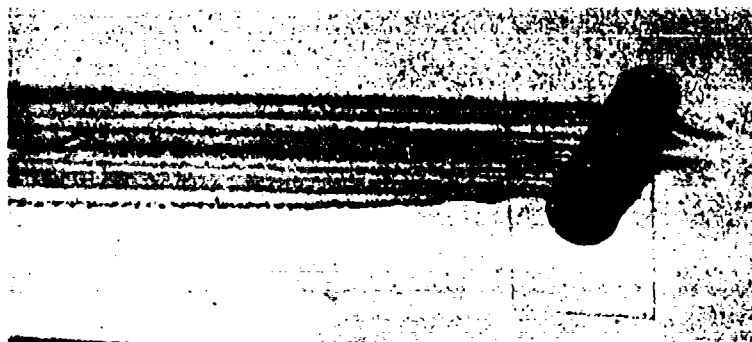


Fig. 4 c. View of dye from cylinder moving ahead of cylinder to delineate ultimately a vortex pair  $R = 40$  and  $F_d = 0.16$ .

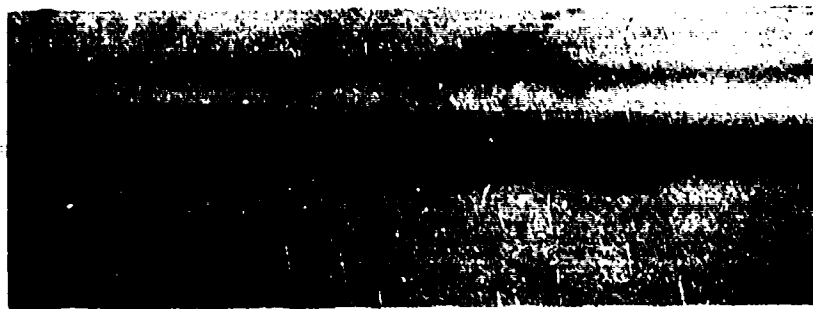


Fig. 5 a. Upstream vortices and small lee waves.  $R = 77$  and  $F_d = 0.31$ .



Fig. 5 b. Cylinder free of upstream and downstream vortices. Lee waves present.  $R = 62$  and  $F_d = 0.248$ .



Fig. 5 c. Cylinder with attached, stable, downstream vortices with lee waves.  $R = 160$  and  $F_d = 0.641$ .



Fig. 5 d. Karman vortex street decaying downstream of cylinder.  $R = 444$  and  $F_d = 1.78$ .

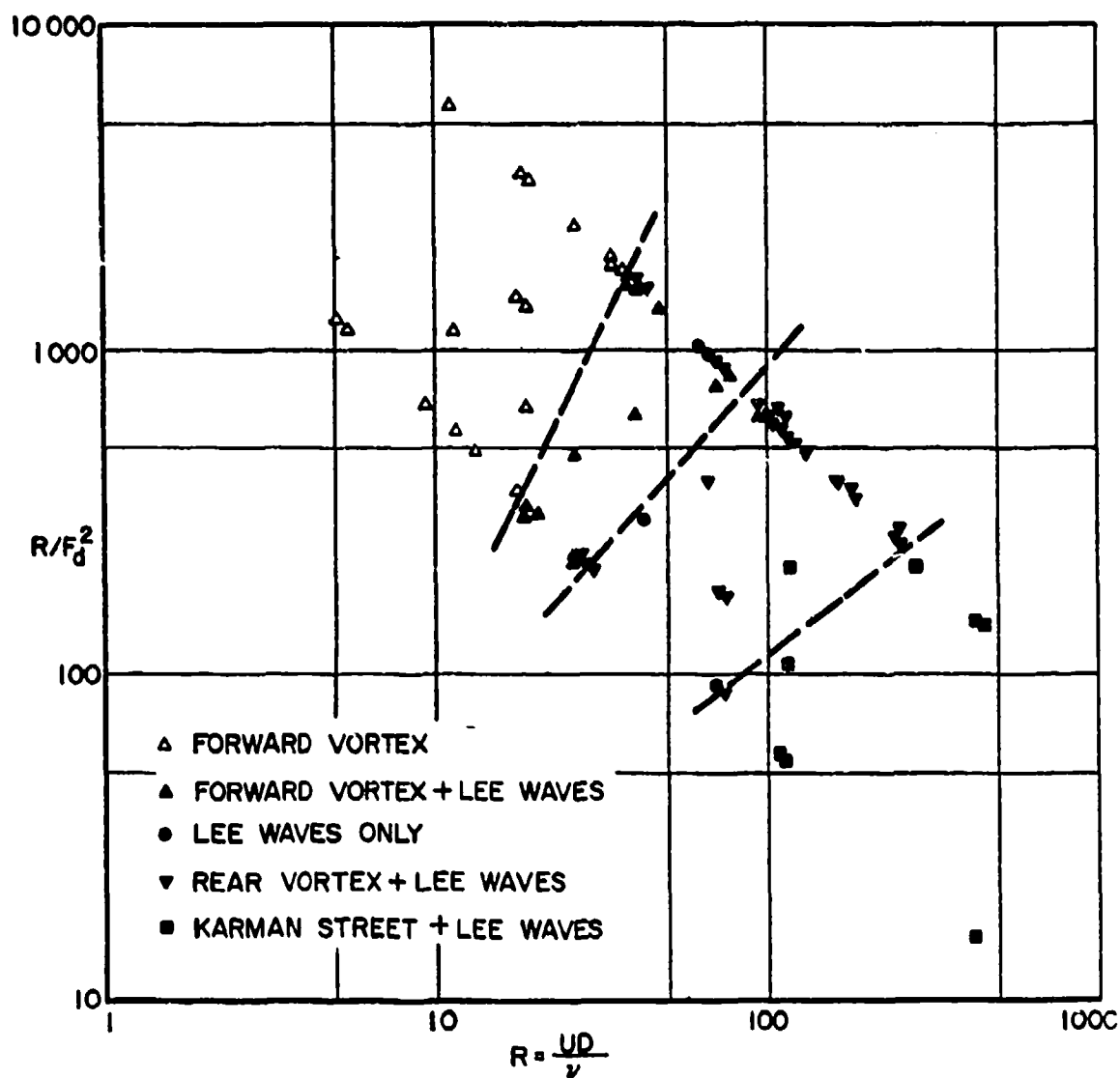


Fig. 6 Flow phenomena diagram for two dimensional circular cylinder translating in a stratified fluid. Vertical coordinate is ratio of the buoyant and viscous forces.

encountered when towing a two dimensional cylinder through a stratified fluid. In order to map these phenomena with the aid of the flow parameters, figure 6 was constructed. The vertical coordinate was selected because it represents the ratio of the buoyant to the viscous forces. These two forces in turn govern the motion. The lowest ordinate, almost a homogeneous fluid, is also a helpful guidepost because of Taneda's work (1956). Suggested flow-regime boundaries are included.

#### Experiments with a vertical plate and a sphere

These results are being included because of their general pertinence to what has just been presented. The flat plate was towed at one density gradient and four different speeds. Figure 7 shows that for  $R = 9.3$  and  $F_D = 0.037$  the liquid followed directly behind the plate. The downstream blocking is on the verge of ceasing with  $R = 27$  and  $F_D = 0.108$ . With  $R = 69$  and  $F_D = 0.275$  a slight nose vortex was observed as well as an apparently stable, attached, downstream vortex pair. A possibly turbulent, but confined, wake occurs for  $R = 115$  and  $F_D = 0.461$ . Lee waves were also present.

Up until now the experiments have demonstrated that density stratification can inhibit the flow over bodies, create lee waves, delay separation and confine turbulent wakes. What will be the effect of stratification on the flow past a sphere where the possibility exists for the fluid to move around the body, an option not applicable to the two dimensional body? An abbreviated series of experiments were conducted to explore this question. The density gradient was fixed at 0.4 or 0.08 mg./cm.<sup>4</sup> and the speed was varied. At the lowest speed the flow as visualized with the aid of the dye crystals on the sphere, almost all of the liquid flowed around the sphere. The dye was swept off the sphere in a vertical sheet that was straight, aligned with the direction of the sphere's motion and bisected the sphere. It appeared as a line when viewed from above as is shown in figure 8. As the speed was

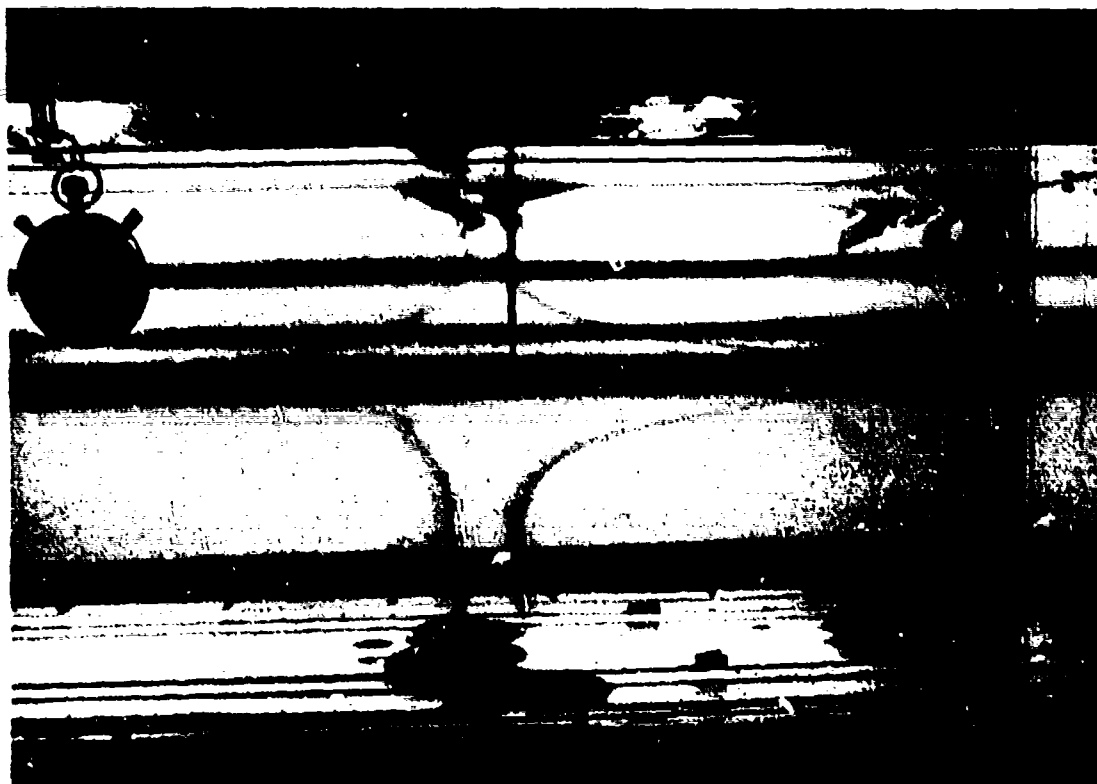


Fig. 7. Vertical plate moving toward the left.  $R = 93$  and  $F_d = 0.037$ . Dye sheets in central channel show up-stream and downstream blocking.





Fig. 8. Sphere being towed in a stratified liquid  $R = 17.6$  and  $F_d = 0.037$ . Wake appears two dimensional in view. Extent of latitudes having horizontal flow can be inferred.

increased with each trial, the vertical extent over which the liquid passed around the sphere, instead of over and under it, decreased. The sheet-like wake also showed some two dimensional instabilities. Figure 9 shows such a wavy wake at several times while the sphere is translating. The similarity between these pictures and those of Homann (1936), which also can be seen in Schlichting (1955), is interesting. It is judged on the basis of the limited tests that for the density gradient used the flow became axisymmetric for  $Re$  greater than 140. Additional experiments are currently underway.

### Conclusions

An initial sally into the realm of flows induced by bodies moving in a stratified fluid has proved interesting and informative. For a given value of the ratio between the buoyant and viscous forces a series of flows past a cylinder occur with increasing Reynolds number. At the lowest Reynolds numbers there is blocking, upstream and downstream, with nose vortices on the body. Increased Reynolds numbers result in the elimination of downstream blocking and a decrease in the size of the nose vortices. Sharply attenuated lee waves begin to occur. The nose vortices disappear and shortly thereafter a stable vortex pair attached to the rear of the cylinder appears. The lee waves continue and grow in extent. Ultimately the vortices become unstable and are shed from the cylinder yielding a Karman vortex street. This wake becomes increasingly irregular with increased Reynolds number. Still, its extent is confined and ultimately reduced downstream of the cylinder. The experiments were restricted to Reynolds numbers less than 500. As the ratio of the buoyant forces to the viscous forces was increased the aforementioned flow transitions occurred at even higher Reynolds number.

The one set of experiments for the flow past a vertical flat plate showed transitions similar to those just mentioned. For sufficiently large density gradients the flow past a sphere, when viewed



Fig. 9. Four stages in the development of a two dimensional Karman vortex street behind a sphere moving in a stratified liquid.  $R = 377$  and  $F_v = 0.8$ .

from above, appears similar to the two dimensional flow past a cylinder in a homogeneous fluid. This is to say that the fluid moves around the sphere over a major portion of its height instead of flowing over and under it.

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